8. Consider the basketball shooting game, define success rate as number of successful shoots divided by number of total shoots. Assume the successful rate rising from below 0.5 to above 0.5, is there a moment which has exactly success rate 0.5.

There is a moment which has exactly success rate 0.5.

For example, we use 0 to denote Failure and 1 denote Success. The sequence is 0, 1, 0, 1, 1….

The first three shooting rate is 1/3, the fourth rate is 1/2, and the fifth rate is 3/5.

Proof: Denote success by S and Failure by F.

When S < F, the successful rate < ½

S = F, the successful rate = ½

S > F, the successful rate > ½

If at the nth shoot the successful rate first > ½, the last shooting must be a successful shoot. It means S > F. Since S and F are both integers, we have S >= F + 1 and S -1 >= F.

S – 1 >= F means if we remove the last successful shoot at time n, we must have S = F at time n-1. Otherwise at time n-1 we have S > F, and it won’t be the first time successful rate > ½.

It would be the same if the shooting rate is 80%. Then S = 4F.

4. (sellside) Given a stock, assume the implied volatility to time t1 is σ1 and the implied volatility to time t2 is σ2, calculate the correlation between St1 and St2 .

Assume the stock price follows Brownian motion, the variance of change in stock price Var(S2 – S1) = t2 –t1.